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Short Communication

Transverse vibration of thin solid and annular circular plate with attached discrete masses

Vinayak Ranjan*, M.K. Ghosh

Department of Mechanical Engineering, Institute of Technology, Banaras Hindu University, Varanasi, 221005, India

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1. Introduction

Circular plates have many engineering applications. These are commonly found in spacecrafts, missiles, land based vehicles, underwater vessels and structures. In many situations, these plates carry loads at eccentric positions. Therefore, one must know the fundamental frequencies of the plates with different boundary conditions with point masses placed at any arbitrary position. Leissa [1] and Laura et al. [2] have reviewed analytically most of the early works in the area of free vibration of circular plate with concentrated mass attached at the centre of the plate. Bambill et al. [3] determined the fundamental frequency of vibration of circular plate carrying a concentrated mass at an arbitrary position with different boundary conditions using variational approach with Rayleigh Ritz formulation.

The present study is concerned with the determination of the fundamental frequency of vibration of solid and annular circular plate with different boundary conditions and with point mass attached at an arbitrary position using finite element analysis (FEA). The fundamental frequency of vibration of clamped and simply supported solid circular plate with point mass attached at the centre determined by other researchers [1–3] are compared with the present study. A good agreement is obtained in the values of fundamental frequency. This methodology has been extended to determine the fundamental frequency of solid and annular plate with different boundary condition with varying point masses placed at an arbitrary position.

2. Modelling procedure and numerical results

Finite element method has been employed for modelling the vibration response of the plate, wherein a system of equations is constructed which is solved as an eigenvalue/eigenvector problem. According to the Finite Element method, the general equation of motion for the free vibration of the plate system is given by

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\}, \tag{1}$$

where $[M]$ is the structural mass matrix, $[K]$ the structural stiffness matrix, $\{\ddot{q}\}$ the nodal acceleration vector, and $\{q\}$ the nodal displacement vector.

*Corresponding author. Scientist (NIRJAFT), Indian Council of Agricultural Research, 12 Regent Park, Kolkata, 700040, India.
E-mail addresses: vinayakranjan@yahoo.com (V. Ranjan), mkghosh47@yahoo.com (M.K. Ghosh).

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Eq. (1) is used to determine the mode shapes, also called eigenvector and the natural frequencies of the plate system. ANSYS 5.4 codes are used for computational analysis. The basic system consists of a thin solid circular plate of 0.3 m radius (R) and 0.01 m thickness (h). The Young modulus (E), the density (ρ) and the Poisson's ratio (v) of the plate material are 206.8 GPa, 7820 kg/m^3 and 0.3, respectively. The point masses are considered rigidly attached to the plate. The four node shell element [shell 63 in ANSYS 5.4] having six degrees of freedom at each node, three for translational in x, y, z directions and three for rotational displacements about x, y, z direction is used to model the plate. The solid plate is divided into 624 elements.

The dimensionless fundamental frequency coefficients λ are calculated as

$$\lambda = 2\pi f \left\lfloor \sqrt{\rho h / D} \right\rfloor R^2,$$

where f is the natural frequency in Hz and D the flexural rigidity $= Eh^3/[12(1-v^2)]$.

The boundary conditions are

$$\mathrm{UX} = \mathrm{UY} = \mathrm{UZ} = \mathrm{ROTX} = \mathrm{ROTY} = \mathrm{ROTZ} = 0 \text{ (for clamped plate)},$$

$$\mathrm{UX} = \mathrm{UY} = \mathrm{UZ} = 0 \text{ (for simply supported plate)},$$

where UX, UY, YZ represent the translational degree of freedom along x, y, z axis, respectively, and ROTX, ROTY, ROTZ are rotational degrees of freedom about x, y, z axis, respectively.

The fundamental vibrating coefficients λ for a rigidly clamped solid circular plate carrying concentrated mass at centre is presented in Table 1. The first column of coefficients have been taken from Leissa [1], the second column from Laura et al. [2], third column from Bambill et al. [3], and fourth column depicts the results obtained in the present work. Table 2 presents the fundamental frequency coefficients λ for simply supported solid circular plate with concentrated mass at the centre of the plate. In both the cases, a good agreement is observed between the values obtained by other researchers and the present work obtained through FEA. This methodology has been extended to find out fundamental vibrating coefficients λ for solid and annular plate with different boundary conditions with M/M_p ($M =$ point mass, $M_p =$ plate mass) ratio varying from 0.05 to 2.0. Tables 3 and 4 depict the values of λ for the radial position of the concentrated mass varying from the centre of the solid plate to $0.9 \times R$ while Tables 5 and 6 present the values of λ for solid circular plate with centrally rigidly fixed circular area of $0.1 \times R$ radius and $0.2 \times R$ radius, respectively. The radial position of the concentrated mass varies from $0.2 \times R$ and $0.3 \times R$ in Tables 5 and 6, respectively, to the

Table 1
Frequency coefficients $\lambda = 2\pi f \left\lfloor \sqrt{\rho h / D} \right\rfloor R^2$ for a rigidly clamped circular plate carrying a concentrated mass at its centre

M/M_p	Leissa [1]	Laura et al. [2]	Bambill et al. [3]	Present study
0.00	10.214	10.22	10.226	10.135
0.05	9.012	9.01	9.012	8.938
0.10	8.111	8.11	8.111	8.045
0.20	7.000	6.87	6.872	6.816
0.50	5.000	5.02	5.023	4.983
1.00	3.750	3.75	3.759	3.730

Table 2
Frequency coefficients $\lambda = 2\pi f \left\lfloor \sqrt{\rho h / D} \right\rfloor R^2$ for a simply supported circular plate carrying a concentrated mass at its centre

M/M_p	Leissa [1]	Laura et al. [2]	Bambill et al. [3]	Present study
0.00	4.935	4.93	4.936	4.898
0.05	—	—	4.547	4.512
0.10	—	4.23	4.231	4.200
0.20	3.767	3.75	3.750	3.722
0.50	2.945	2.92	2.913	2.892
1.00	2.291	2.25	2.255	2.239

Table 3
Frequency fundamental coefficients $\lambda = 2\pi f \lfloor \sqrt{\rho h/D} \rfloor R^2$ for a clamped solid circular plate carrying a concentrated mass M at $(r \times R, \theta)$ position

Mass position $(r \times R, \theta)$	M/M_p							
r	0.05	0.10	0.20	0.30	0.50	1.0	1.5	2.0
0	8.938	8.045	6.816	6.005	4.983	3.730	3.103	2.719
0.1	8.977	8.098	6.873	6.060	5.019	3.769	3.140	2.748
0.2	9.091	8.255	7.049	6.231	5.184	3.887	3.240	2.835
0.3	9.270	8.517	7.354	6.531	5.453	4.098	3.418	2.992
0.4	9.494	8.874	7.807	6.992	5.876	4.433	3.700	3.240
0.5	9.729	9.291	8.420	7.655	6.514	4.950	4.138	3.626
0.6	9.928	9.691	9.140	8.546	7.470	5.769	4.840	4.246
0.7	10.058	9.970	9.757	9.489	8.811	7.151	6.059	5.334
0.8	10.117	10.099	10.058	10.009	9.886	9.331	8.455	7.611
0.9	10.133	10.132	10.130	10.127	10.121	10.106	10.088	10.065

Table 4
Frequency fundamental coefficients $\lambda = 2\pi f \lfloor \sqrt{\rho h/D} \rfloor R^2$ for a simply supported solid circular plate carrying a concentrated mass M at $(r \times R, \theta)$ position

Mass position $(r \times R, \theta)$	M/M_p							
r	0.05	0.10	0.20	0.30	0.50	1.0	1.5	2.0
0	4.51	4.20	3.72	3.37	2.89	2.24	1.89	1.67
0.1	4.52	4.21	3.74	3.39	2.91	2.25	1.90	1.68
0.2	4.55	4.27	3.79	3.45	2.97	2.30	1.95	1.72
0.3	4.59	4.32	3.88	3.54	3.06	2.39	2.02	1.79
0.4	4.64	4.40	4.00	3.68	3.21	2.52	2.14	1.89
0.5	4.70	4.51	4.17	3.88	3.42	2.72	2.32	2.05
0.6	4.76	4.62	4.36	4.12	3.17	3.00	2.59	2.30
0.7	4.82	4.73	4.56	4.40	4.01	3.44	3.00	2.69
0.8	4.86	4.82	4.74	4.66	4.49	4.06	3.68	3.37
0.9	4.89	4.88	4.86	4.84	4.80	4.69	4.57	4.43

Table 5
Frequency fundamental coefficients $\lambda = 2\pi f \lfloor \sqrt{\rho h/D} \rfloor R^2$ for a solid circular plate carrying a concentrated mass M at $(r \times R, \theta)$ position and with centrally rigidly fixed circular area of $0.1 \times R$ radius

Mass position $(r \times R, \theta)$	M/M_p							
r	0.05	0.10	0.20	0.30	0.50	1.0	1.5	2.0
0.2	3.594	3.593	3.591	3.590	3.586	3.577	3.568	3.558
0.3	3.587	3.580	3.563	3.550	3.511	3.415	3.310	3.202
0.4	3.569	3.541	3.484	3.424	3.300	3.000	2.741	2.528
0.5	3.535	3.471	3.338	3.207	2.963	2.500	2.191	1.970
0.6	3.482	3.364	3.131	2.924	2.592	2.074	1.774	1.575
0.7	3.411	3.222	2.888	2.625	2.251	1.738	1.465	1.290
0.8	3.321	3.054	2.635	2.340	1.955	1.473	1.230	1.077
0.9	3.211	2.866	2.384	2.077	1.702	1.259	1.044	0.912
1.0	3.083	2.664	2.145	1.837	1.483	1.083	0.894	0.778

free outer boundary. In Tables 7 and 8, the concentrated mass is fixed at the free outer boundary of the annular plate.

The mass of the plate M_p for annular plate becomes $\pi R^2 \rho h(1 - r^2)$.

Table 6
Frequency fundamental coefficients $\lambda = 2\pi f\left\lfloor\sqrt{\rho h/D}\right\rfloor R^2$ for a solid circular plate carrying a concentrated mass M at ($r \times R$, θ) position and with centrally rigidly fixed circular area of $0.2 \times R$ radius

Mass position ($r \times R$, θ)	M/M_p							
r	0.05	0.10	0.20	0.30	0.50	1.0	1.5	2.0
0.3	4.96	4.96	4.96	4.96	4.95	4.94	4.92	4.90
0.4	4.95	4.94	4.90	4.86	4.78	4.54	4.28	4.03
0.5	4.91	4.85	4.71	4.55	4.26	3.65	3.22	2.90
0.6	4.83	4.67	4.34	4.05	3.58	2.86	2.44	2.17
0.7	4.69	4.40	3.89	3.51	2.98	2.29	1.92	1.69
0.8	4.51	4.07	3.44	3.02	2.50	1.87	1.55	1.36
0.9	4.28	3.72	3.02	2.60	2.11	1.55	1.28	1.11
1.0	4.02	3.36	2.74	2.23	1.79	1.29	1.07	0.93

Table 7
Frequency fundamental coefficients $\lambda = 2\pi f\left\lfloor\sqrt{\rho h/D}\right\rfloor R^2$ for an annular plate carrying a concentrated mass M at outer free boundary (R, θ) and rigidly clamped at inner boundary

Inner boundary position ($r \times R$)	M/M_p							
r	0.05	0.10	0.20	0.30	0.50	1.0	1.5	2.0
0.1	3.08	2.66	2.14	1.84	1.48	1.08	0.89	0.78
0.2	4.06	3.40	2.68	2.27	1.82	1.32	1.09	0.94
0.3	5.25	4.32	3.28	2.82	2.24	1.62	1.30	1016
0.4	6.96	5.61	4.28	3.59	2.84	2.04	1.68	1.46
0.5	9.62	7.59	5.71	4.76	3.76	2.69	2.21	1.92
0.6	14.15	10.93	8.11	6.74	5.29	3.78	3.10	2.69
0.7	23.11	17.49	12.83	10.61	8.31	5.92	4.84	4.20
0.8	46.92	35.00	25.46	20.99	16.38	11.65	9.53	8.27

Table 8
Frequency fundamental coefficients $\lambda = 2\pi f\left\lfloor\sqrt{\rho h/D}\right\rfloor R^2$ for an annular plate carrying a concentrated mass M at outer free boundary (R, θ) and simply supported at inner boundary

Inner boundary position ($r \times R$)	M/M_p							
r	0.05	0.10	0.20	0.30	0.50	1.0	1.5	2.0
0.1	2.31	2.06	1.71	1.48	1.21	0.89	0.74	0.65
0.2	2.69	2.35	1.92	1.65	1.34	0.99	0.82	0.71
0.3	3.03	2.63	2.09	1.83	1.48	1.09	0.88	0.78
0.4	3.38	2.95	2.39	2.06	1.66	1.22	1.01	0.88
0.5	3.86	3.39	2.76	2.37	1.92	1.40	1.16	1.01
0.6	4.61	4.08	3.32	2.86	2.31	1.69	1.39	1.21
0.7	5.94	5.26	4.29	3.68	2.98	2.17	1.79	1.56
0.8	8.76	7.72	6.26	5.37	4.33	3.15	2.60	2.2

3. Conclusions

Fundamental frequency coefficients of clamped and simply supported solid circular plate with point mass placed at the centre obtained by other researcher are in agreement with the present work using finite element method. Using this method, the fundamental frequency coefficients of solid plate with point mass placed at arbitrary positions and of annular plate with point mass placed at outer free boundary with different boundary conditions are presented.

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